COMPUTER CALCULATION OF THE TEMPERATURE DISTRIBUTION FOR A STEEL CASTING SOLIDIFYING IN A MOLD

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An algorithm and program have been developed for handling a two-dimensional axially symmetrical problem for thermal conduction in an inhomogeneous region with time-varying boundary conditions; the material releases a latent heat of crystallization over a temperature range, and the thermophysical characteristics are dependent on temperature.

There are several papers on computer calculation of two-dimensional crystallization processes; an example has been given of a Peaceman—Rackford solution to a thermal conduction problem [1]. Some results were presented on electroslag melting. The heat released in crystallization has also been incorporated [2], while the equations of thermal conduction were solved by finite differences with an explicit scheme containing a set time step subject to restrictions. The present study does not involve such restriction.

The thermal processes in cooling and solidification in a metal mold are considered as a two-dimensional axially symmetric thermal conduction problem; the actual sizes, shape, and design of the mold are incorporated, together with the various cooling conditions for the different parts of the casting and mold. The heat transfer in the gap between the casting and mold is reproduced by introducing variable thermal resistances. The thermophysical characteristics of the materials can vary with temperature, while the latent heat may be released over a certain temperature range, and the surrounding temperature may alter.

We assume that the mold is filled instantaneously with the liquid metal; it is assumed that there is a certain given temperature distribution in the mold, support, and casting before cooling starts. The outside and inside surfaces of the body may be conical or cylindrical. The mold has a two-layer jacket, with the inner layer of heat-resisting material and the outer a metal jacket. There may be any relationship between the sizes of the mold, the support, and the jacket, which allows one to examine castings of various shapes and weights.

Figure 1 shows the system. There are three regions with different thermophysical characteristics: the casting (given quantities $c\gamma_S(t°)$, $\lambda_S(t°)$, Q, t°_p, t°_l, t°_S), the mold, the support, and the jacket in the cover ($c\gamma_M(t°)$, $\lambda_M(t°)$, t°_{in}), and also the refractory lining ($c\gamma_r(t°)$, $\lambda_r(t°)$, t°_{in}).

The specific heat $c\gamma$ and thermal conductivity λ are dependent on temperature and are given as tables; the effective thermal conductivities incorporate the convection currents in the liquid. The heat-transfer coefficients α_{eg}, α_{pc}, α_{cl}, and α_{lg} incorporate the heat transfer between the mold and the support (line eg), between the casting and the lining (on lines pc and cl), and between the casting and the mold (on line lg, Fig. 1).

The loss of heat to the surroundings is incorporated via the heat-transfer coefficients α_{ex}, α_{sid}, α_{ad}; the heat transfer from the upper surface along the line ex is represented by α_{ex}, which also incorporates the thermal resistance of the layer of material δ_p having thermal conductivity λ_p:

$$\alpha_{ex}=\frac{\lambda_p}{\delta_p}. \tag{1}$$

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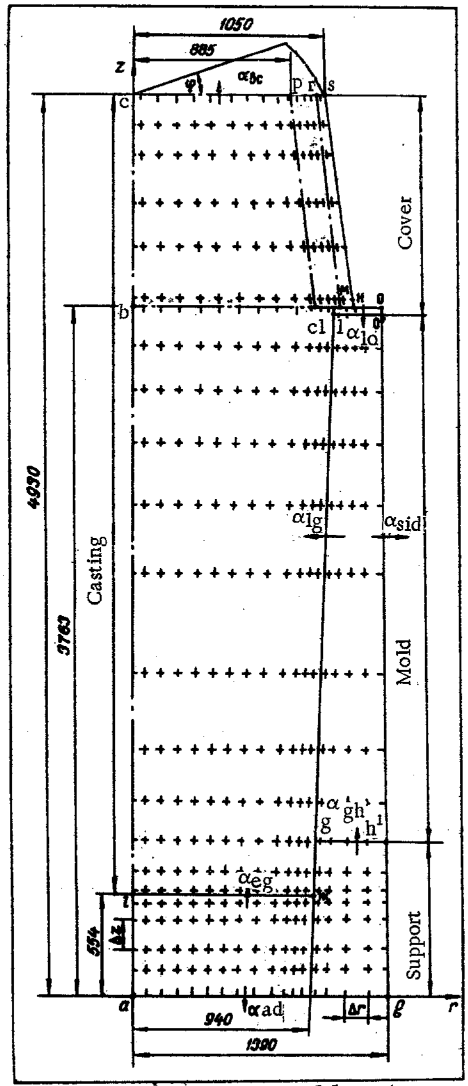

Fig. 1. The casting-mold system.

The heat transfer at the interfaces between the mold and the support and jacket are incorporated via additional thermal contact resistances. The heat-transfer coefficient α_{cm} incorporates ideal thermal contact initially after pouring, when no reasonably solid crust has formed, and hence there is no gap between the mold and the casting. When a gap has been produced, the coefficient is calculated from the following formula, which incorporates radiation and convection:

$$\alpha_{cm} = \alpha_{rad} + \alpha_{conv} + \alpha_{cond} \tag{2}$$

$$\alpha_{rad} = \frac{k\left(\frac{t_1}{100}\right)^4 - \left(\frac{t_2}{100}\right)^4}{t_1 - t_2}, \tag{3}$$

$$k = \frac{1}{\frac{1}{k_1} + \frac{F_1}{F_2}\left(\frac{1}{k_2} - \frac{1}{k_0}\right)}, \tag{4}$$

where k_1 and k_2 are the emissivities of the casting, mold, and absolutely black body, $k_0 = 5.6\ \mathrm{J/m^3 \cdot sec \cdot {}^\circ K}$.

The latent heat of crystallization Q(t) follows an arbitrary mode of release, and the heat release over the range $(t_l - t_s)$ is incorporated in Δc; if Q(t) is linear,

$$\Delta c = \frac{Q}{t_1 - t_s}. \tag{5}$$

The system is divided into elementary volumes Δr along the radius and Δz in the height; the nodal points at which the temperature was calculated lay at the corners of the elements. The distributed heat couplings in a real body were replaced by discrete conductances between the nodal points (Fig. 2). These conductances were calculated from the following formulas: in the radial direction

$$\Lambda_{ij} = \frac{(\lambda_i + \lambda_j)(z_e - z_m)(r_i + r_j)}{8(r_j - r_i)}, \tag{6}$$

along the z axis

$$\Lambda_{ej} = \frac{(\lambda_e + \lambda_j)(r_f + r_k - r_d - r_i)(r_j + r_e)}{16(z_e - z_j)}. \tag{7}$$

The adjoint thermal capacity at each node was

$$\Lambda_{jj} = \frac{(r_k - r_i)(z_e - z_m)}{4\Delta\tau} c\gamma r_j. \tag{8}$$

Consider the conditions for heat balance at some point j (Fig. 2); if at instant τ_{m-1} the temperature at known j was t_j^{m-1}, while after a time $\Delta\tau$, at the instant $\tau_m = \tau_{m-1} + \Delta\tau$, it becomes t_j^m, then the heat content at this node has altered by

$$\Delta c = c_j (t_j^{m-1} - t_j^m). \tag{9}$$

This change is compensated by the heat flowing in from adjacent nodes on account of the difference between the temperature at node j and the temperatures at surrounding nodes i, k, e, and m; if the heat fluxes from the adjacent nodes are denoted by q_{ij}, q_{kj}, q_{ej}, q_{mj}, then the heat-balance equation for a node will take the form

$$q_{ij} + q_{kj} + q_{ej} + q_{mj} + \frac{c_j}{\Delta\tau}(t_j^{m-1} - t_j^m) = 0. \tag{10}$$

We express the heat fluxes via the temperature differences and conductances, to get

$$\Lambda_{ij}(t_i^m - t_j^m) + \Lambda_{kj}(t_k^m - t_j^m) + \Lambda_{ej}(t_e^m - t_j^m) + \Lambda_{mj}(t_m^m - t_j^m) + \frac{c_j}{\Delta\tau}(t_j^{m-1} - t_j^m) = 0, \tag{11}$$

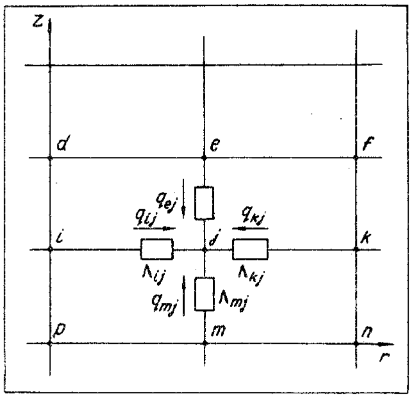

Fig. 2. Nodal point with discrete conductances.

and denote $c_j/\Delta\tau$ by Λ_{jj} and rearrange the terms to get

$$\Lambda_{ij}t_i^m + \Lambda_{kj}t_k^m + \Lambda_{ej}t_e^m + \Lambda_{mj}t_m^m - (\Lambda_{ij} + \Lambda_{kj} + \Lambda_{ej} + \Lambda_{mj} + \Lambda_{jj})\, t_j^m = -\Lambda_{jj}t_j^{m-1}.$$

Similar equations can be drawn up for all the nodes. The complete system for all the nodes reflects the coupling between the temperatures at the nodes at the previous instant τ_{m-1} and the next instance τ_m. The unknowns are t_i^m, t_k^m, t_m^m, t_j^m ... and are the temperatures at the subsequent instant τ_m, together with the coefficients Λ_{ij}, Λ_{kj} ..., which represent certain thermal conductivities, while the constant terms $(-\Lambda_{jj}t_j^{m-1})$ may be defined from the temperatures at the corresponding points at time τ_{m-1}. The solution gives the temperature distribution at the nodes at instant

$$\tau_m = \tau_{m-1} + \Delta\tau.$$

This method enables one to use large time steps $\Delta\tau$ without restricting the size of the elements, since the system of equations always satisfies the conditions for heat balance. This results in considerable acceleration relative to computations via the explicit scheme, in which the process becomes unstable, if the step exceeds a certain size.

The thermophysical characteristics as functions of temperature are incorporated by determining all the Λ afresh in accordance with the relationships for $c\gamma$, λ, and α as functions of temperature after the system has been solved each time; the temperature change in the surroundings is incorporated in each iteration in deriving the constant term.

The coefficient matrix has a strip structure and is symmetrical, which results in considerable economy in machine store.

An Algol program has been written for the M220 computer; before the machine handles the problem, the following information must be input: the initial temperature distribution $t_{ij}(r, z)$, the surrounding temperature $t_{sur}(\tau)$ as a function of time, the heat-transfer coefficients between the casting and the mold α_{cm} and the air α_{air}, tables for $\lambda(t)$ and $c\gamma(t)$, the temperatures for onset and completion of crystallization t_l and t_s, and the latent heat of crystallization Q. The sizes of casting and mold are defined by supplying the radii r_i on the lines ad, bo, and cs, while the heights are divided into elementary volumes; also logic conditions are supplied that reflect the disposition of the various parts.

Figure 3 shows the block diagram for the process; in step 2, the store receives the initial temperature distribution; in a subsequent step 3, the radii r_j are calculated, together with the heights z_j. In step 4 the thermophysical parameters are selected from the table in accordance with the temperature. If the current temperature lies in the range t_l–t_s, then one incorporates the latent-heat release in steps 5 and 6. The heat-transfer coefficients are selected in 8 in accordance with the temperature or the current time. Then the discrete thermal conductances are calculated, in steps 10-12, and finally the system is solved by Gauss's method.

Figure 4 shows the solution for the temperature in a casting of weight 100 tons; the initial data were the surrounding temperature t_a = 50°C, initial temperatures of mold t_M = 150°C, casting t_c = 1500°C, liquidus temperature t_l = 1500°C, solidus t_s = 1450°C, latent heat Q = 201.12 · 10^7 J/m^3, specific heat of cast iron γ_C = 6950 kg/m^3, specific heat of firebrick γ_{fi} = 1800 kg/m^3, thermal conductivity of cast iron λ_c = 37.2 J/m · sec · deg, the same for firebrick λ_f = 0.832 + 0.00058 t° J/m · sec · deg, specific heat of cast iron c_C = 628.5 J/kg · deg, the same for firebrick c_{fi} = 878 + 0.00065 t° J/kg · deg, heat-transfer coefficient for the upper surface of the casting with the air α_{upp} = 46.5 J/m^2 · sec · deg, and thermal capacity and thermal conductivity of steel as in Table 1, with the heat-transfer coefficients for the lower surface as in Table 2, the heat-transfer coefficient between casting and mold as in Table 3, together with the same for the side surface and the working time step, while the casting size is given in Fig. 1.

To obtain the maximum accuracy with a restricted number of elements, one always uses in the two-dimensional case a nonuniform division in radius and height; calculations on castings have previously been performed with 22 steps along the radius and 34 in height. Improved methods and accumulated experience have enabled us now to obtain good accuracy with 16 steps along the radius and 22 in height. In that case one does not need to use the M-220 tape store, while the running time for 1 time step is 2 min.

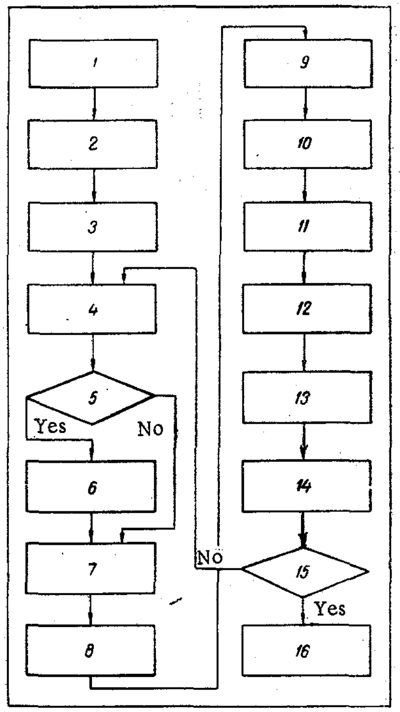

Fig. 3

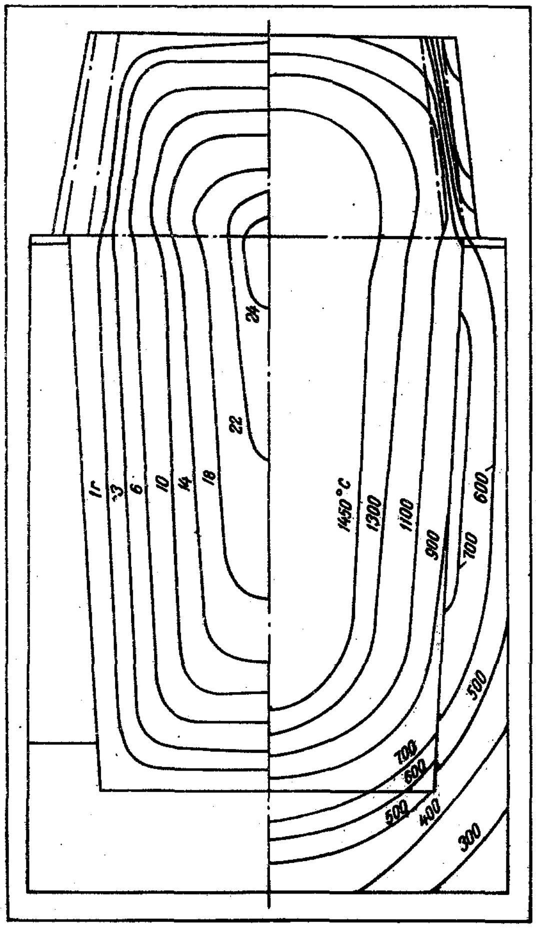

Fig. 4

Fig. 3. Block diagram of the calculation scheme: 1) input data (10 → 2); 2) initial temperature distribution input; 3) calculation of R and dR for layers; 4) table readout and interpolation for $\gamma(T^\circ)$ and $\lambda(T^\circ)$ for casting; 5) test for $T_s \leq T \leq T_l$; 6) latent heat release $\Delta\Theta_{lat}$; 7) table readout and interpolation for $\gamma(T^\circ)$ and $\lambda(T^\circ)$ for mold; 8) table readout and interpolation for α_{ex}, α_{ad}, α_{sid}, α_{ps}, α_{lg}, and α_{eg}; 9) table readout and interpolation for T (°C) of surroundings; 10) calculation of $\Lambda_{i,i+1}$, $\Lambda_{j,j}$, $\Lambda_{i,j}$; 11) calculation of Λ_{sp-m} and Λ_{surf}; 12) formulation of the matrix elements for the system of algebraic equations; 13) solution of system of linear algebraic equations; 14) printout of T°, current time, and other parameters; 15) test for end, $\tau_m < T_F^\circ$; $T^\circ < \tau_F$ 16) halt.

Fig. 4. Movement of the solidus line and distribution of the temperature (°C) at 10 h after pouring.

The scope for using a large time step enables one to calculate the complete solidification in 30-40 steps; at the start, the step can be a fraction of a minute, increasing subsequently to several hours.

At the same time, the same method was used to write a program for one-dimensional axially symmetrical cases. This program in the one-dimensional form reflects all the above heat-transfer conditions. It is also possible to use a very detailed division in that case along the radius and small time steps, which provide very high accuracy.

Comparison with two-dimensional calculations has shown that the one-dimensional calculation closely reproduces the temperature distribution along the radius at the mid point in the height, so the one-dimensional program can be used in preliminary analysis before handling a new two-dimensional problem. When the two-dimensional case has to be considered, the restrictions on store size and running time do not allow one to use very small steps, so each new casting is considered by solving the one-dimensional problems to determine the minimum number of elements required in the two-dimensional region to provide the required accuracy.

TABLE 1. Temperature Dependence of $c\gamma_c$ and λ_c (Casting)

t, °C	0	20	1200	1449	1450	1500	1600
$c\gamma_c$, J/m^3·deg	3618070	3618070	5028000	5185130	5185130	5866000	5866000
λ_c, J/m·sec·deg	29,1	25,6	22,8	21,65	20,02	18,62	17,81

TABLE 2. Temperature Dependence of α_{ad}

t, °C	0	500	700	900	1000	1200	1400	1600
α_{ad}, J/m^2·sec·deg	42,13	42,13	69,14	107,9	132,68	194,4	272,35	370,12

TABLE 3. Dependence of α_{cm}, α_{sid}, and $\Delta\tau_h$ on Working Time

τ, h	0	0,01	0,033	0,1	0,3	1,0	2,0	10,0	30,0
α_{cm}, J/m^2·sec·deg	698,3	346,8	346,8	346,8	258,4	226,9	207,17	181,57	181,57
α_{sid}, J/m^2·sec·deg	14,9	14,9	14,9	14,9	26,77	34,9	38,41	39,57	36,1
$\Delta\tau_w$, h	0,002	0,002	0,00833	0,025	0,075	0,25	0,5	1,0	1,0

Solutions obtained from this algorithm have been compared with the exact solutions due to Lykov [3], and the agreement is good.

Castings of weights 4, 7, 52, and 169 tons were used in experiments; the experimental conditions were reproduced closely on the computer, which provided considerable experience on the fitting of thermophysical characteristics $c\gamma$, λ, and α to provide good agreement with experiment. The surface temperature of the mold as calculated by computer differed from the measured value by not more than 25-30°C.

The two-dimensional calculations were compared with the actual structures of the castings to relate the latter to the temperature distribution during solidification.

This method of calculating the castings enables one to examine the effects of all aspects of the mold design and dimensions on the result, together with the effects of the thermophysical characteristics of the materials and other technical features.

NOTATION

α	is the heat transfer coefficient;
α_{cm}	is the heat transfer coefficient between casting and mold;
α_{rad}, α_{conv}, α_{cond}	are the radiation, convection, and conduction heat transfer coefficients;
Q	is the latent heat of solidification;
t_p	is the pouring temperature;
t_l	is the liquidus temperature;
t_s	is the solidus temperature;
t_{in}	is the initial temperature;
t	is the temperature;
$c\gamma$	is the specific heat;
λ	is the specific thermal conductivity;
Λ	is the thermal conductivity;
q	is the heat flux;
r	is the coordinate along r axis;
z	is the coordinate along z axis;

Δz	is the step along z axis;
Δr	is the step along r axis;
τ	is the current time;
$\Delta \tau$	is the time step;
F_1, F_2	are the surface areas;
c	is the heat capacity;
Δc	is the heat capacity variation;
δ_p	is the thickness of filling layer.

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